# SYSTEM OF EQUATIONS: MATH LESSONS IN CLASSICAL LITERATURE

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## ABSTRACT

The aim of this paper is to illustrate mathematical challenges in classical literature and to show possible tasks in order to integrate classical literature in math lessons. Therefore works from different authors like: Jules Verne, Anton Chekhov, etc… are analyzed in this paper. Additionally, some ideas for further tasks are given in this paper. Most of the problems are just simple mathematical equations and can be solved without using a computer. Nevertheless, the computer program "Mathcad" is used in order to solve the problems and to illustrate the solutions.

**KEYWORDS: classical literature, system of equations, Mathcad**

## INTRODUCTION

The emergence of powerful portable mobile devices - smartphones, tablets, and so on has changed our daily life. We can lie on the couch, riding the bus, fly a plane or sit in a cafe and we still always have a close connection with the world's cultural and scientific life. For example, we can simultaneously read a book, listen to the text of the book, watch films about this book, surfing in the internet in order get to know people's opinion about this book, logging in to a forum to leave our opinion about the book and solve problems ... found in the book. Therefore the boundaries between different parts of our lives start to blur. Poetry becomes a part of science, religion becomes a part poetry, etc… It becomes important to work and teach interdisciplinary. This paper tries to integrate classical literature in math lessons and also math lessons in classical literature.

You can find a lot of mathematical problems in classical literature and fiction. Typically, the problems you can find in these novels are quite simple to solve, just some simple financial calculations or a system of equations. Moreover, there also some difficult and complex mathematical challenges. Most of the problems are just some simple

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mathematical equations and can be solved by using "computers", which have also been available to the heroes in these literary works or at least to the authors of these works: pen and paper. In the following parts mathematical problems in classical literature are going to be presented and analyzed.

**ANTON CHEKHOV´S "THE TUTOR"**

The first problem, which is going to be analyzed, is also the simplest one. It is a mathematical task, found in Chekhov`s short story "The Tutor". The task is the following:

> "If a merchant buys 138 yards of cloth, some of which is black and some blue, for 540 roubles, how many yards of each did he buy if the blue cloth cost 5 roubles a yard and the black cloth 3?' Repeat what I have just said."
> […]
> The tutor looks in the back of the book and finds that the answer is 75 and 63. (Chekhov, 1884)

Such a simple problem can be solved without using a computer or even without a calculator. There are two equations and two unknowns. However, in this work the problem will be solved by using the computer. Therefor the most popular "Super-Calculator"-the program Mathcad will be used. This program can be easily installed on every smartphone or tablet PC. A person can quickly learn how to use this program and will not lose his skills to work with this package after a forced break. However, this program is very powerful, too. It allows solving rather complex mathematical problems. Figure 1 shows how the problem is solved in Mathcad. By using the Mathcad command *"solve"*, it is able to solve linear and non-linear equations.

$$\begin{bmatrix} Blue\ Cloth + Black\ Cloth = 138\ Yard \\ 5\ \frac{Ruble}{Yard} \cdot Blue\ Cloth + 3\ \frac{Ruble}{Yard} \cdot Black\ Cloth = 540\ Ruble \end{bmatrix} \xrightarrow{solve, \begin{bmatrix} Blue\ Cloth \\ Black\ Cloth \end{bmatrix}} [63\,Yard\quad 75\,Yard]$$

*Figure 1: Chekhov`s mathematical problem in Mathcad*

Here the number of equations is equal to the number of unknowns. But in reality, the number of equations and unknowns is usually not equal. Even in literature it is possible to find such more complex problems, like the following examples will show.

**DOSTOYEVSKY`S “THE GAMBLER”**

The mathematical problem, which can be found in Dostoyevsky`s “The Gambler”, is much more difficult than the previous one. It deals with exchange rates of different currencies. In order to understand the problem quotes from different parts of the book have been taken, the problem is not as clearly formulated as the previous one. The following quotes illustrate the problem.

Quote 1

> "But you will soon be in receipt of some," retorted the General, reddening a little as he dived into his writing desk and applied himself to a memorandum book. From it he saw that he had 120 roubles of mine in his keeping.”
> "Let us calculate," he went on. "We must translate these roubles into thalers. Here--take 100 thalers, as a round sum. The rest will be safe in my hands."[..]
> There is what is owing to you, four friedrichs d’or and three florins, according to the reckoning here. (Dostoyevsky, 1867)

Quote 2

> Of course, we began by talking on business matters. Polina seemed furious when I handed her only 700 gulden, for she had thought to receive from Paris, as the proceeds of the pledging of her diamonds, at least 2000 gulden, or even more.[…]
> "That has nothing to do with it. Listen to me. Take these 700 florins, and go and play roulette with them. Win as much for me as you can, for I am badly in need of money."(Dostoyevsky, 1867)

Quote 3

> I began by taking out five friedrichs d`or (fifty gulden) and putting them on the even. (Dostoyevsky, 1867)

Quote 4

> "Yes, I have won twelve thousand florins," replied the old lady. "And then there is all this gold. With it the total ought to come to nearly thirteen thousand. How much is that in Russian money? Six thousand roubles, I think?" However, I calculated that the sum would exceed seven thousand roubles--or, at the present rate of exchange, even eight thousand. (Dostoyevsky, 1867)

Quote 5

> "Oui, Madame," was the croupier's polite reply. "No single stake must exceed four thousand florins. That is the regulation."
> "I cannot, Madame. The largest stake allowed is four thousand gulden." (Dostoyevsky, 1867)

Quote 6

> She was to receive exactly four hundred and twenty friedrichs d`or, that is, four thousand florins and twenty friedrichs d`or. (Dostoyevsky, 1867)

Quote 7

> "Polina," I said, "here are twenty-five thousand florins--fifty thousand francs, or more. Take them, and tomorrow throw them in De Griers' face." (Dostoyevsky, 1867)

With the help of all these quotes the exchange rate against the rubble can be found. Following equations can be set up:

| | | |
|---|---|---|
| 120 ruble | = 100 thalers + 4 friedrichs + 3 florins | (Quote 1) |
| 700 gulden | = 700 florins | (Quote 2) |
| 5 friedrichs | = 50 gulden | (Quote 3) |
| 13.000 florins | = 8.000 ruble | (Quote 4) |
| 4.000 florins | = 4.000 gulden | (Quote 5) |
| 420 friedrichs | = 4.000 florins + 20 friedrichs | (Quote 6) |
| 25.00 florins | = 50.000 francs | (Quote 7) |

If the number of equations (seven) exceeds the number of unknowns (five), then the system of equations is called over determined.
The problem of "The Gambler" and can be solved without a computer, by continually calculating the rates of the individual currencies (1 Guilder = 1 Florin, 1 friedrichs = 10 gulden, etc...) and to evaluate them against the ruble. But like in the first task, the computer will be used to solve the problem.

$$\begin{bmatrix} 120\ ruble = 100\ thaler + 4\ Friedrichs + 3\ florin \\ 700\ gulden = 700\ florin \\ 5\ Friedrichs = 50\ gulden \\ 13000\ florin = 8000\ ruble \\ 4000\ florin = 4000\ gulden \\ 420\ Friedrichs = 4000\ florin + 20\ Friedrichs \\ 25000\ florin = 50000\ franc \end{bmatrix} solve, \begin{bmatrix} thaler \\ Friedrichs \\ florin \\ gulden \\ franc \end{bmatrix} \rightarrow [0.94\,ruble\ \ 6.2\,ruble\ \ 0.62\,ruble\ \ 0.62\,ruble\ \ 0.31\,ruble]$$

*Figure 2 Dostoyevsky`s mathematical problem in Mathcad*

Figure 2 shows the mathematical challenge with its solution in Mathcad. Like in the previous task the "*solve*"-operator is used in order to solve the problem. Though the problem is here much more complicated and therefore the "*float*"-operator is additionally used. By using the "float"-operator it is possible to decide how many significant digits the solution has (here two). As it can be seen by means of the solution one thaler was worth 94 kopecks, one florin or gulden 62 kopecks, one franc 31 kopecks and one friedrich was equal to six rubles and two kopecks. So in this novel "Babulenka" won 7930 rubles in the roulette game (Quote 4) and the general paid the teacher 117,73 rubles (Quote 1).

**JULES VERNE´S "TWENTY THOUSAND LEAGUES UNDER THE SEA"**

Even in Jules Verne`s novel "Twenty Thousand Leagues Under the Sea" one mathematical problem can be found. Here, the mathematical problem deals with size of the "Nautilus"-submarine. The following quote makes the problem clear:

> Here, M. Aronnax, are the several dimensions of the boat you are in. It is an elongated cylinder with conical ends. It is very like a cigar in shape, a shape already adopted in London in several constructions of the same sort. […]
> Its area measures 1011.45 square metres; and its contents 1,500.2 cubic metres; that is to say, when completely immersed it displaces 1500.2 cubic metres of water, or 1500.2 metric tons. (Jules Verne, 1870)

Basically, the submarine`s structure can be described by three geometrical shapes. The main part of the submarine is the cylinder. The other part of the submarine consists of two identical cones, which are placed on the bottom and top of the cylinder. Additionally some information is given about the surface and volume. Therefore it is possible to set up 2 equations, but the problem is described by three unknowns. Figure 3 illustrates the problem.

Again, Mathcad is used in order to solve problem. Like in the previous tasks, the "*solve*"-operator is used.

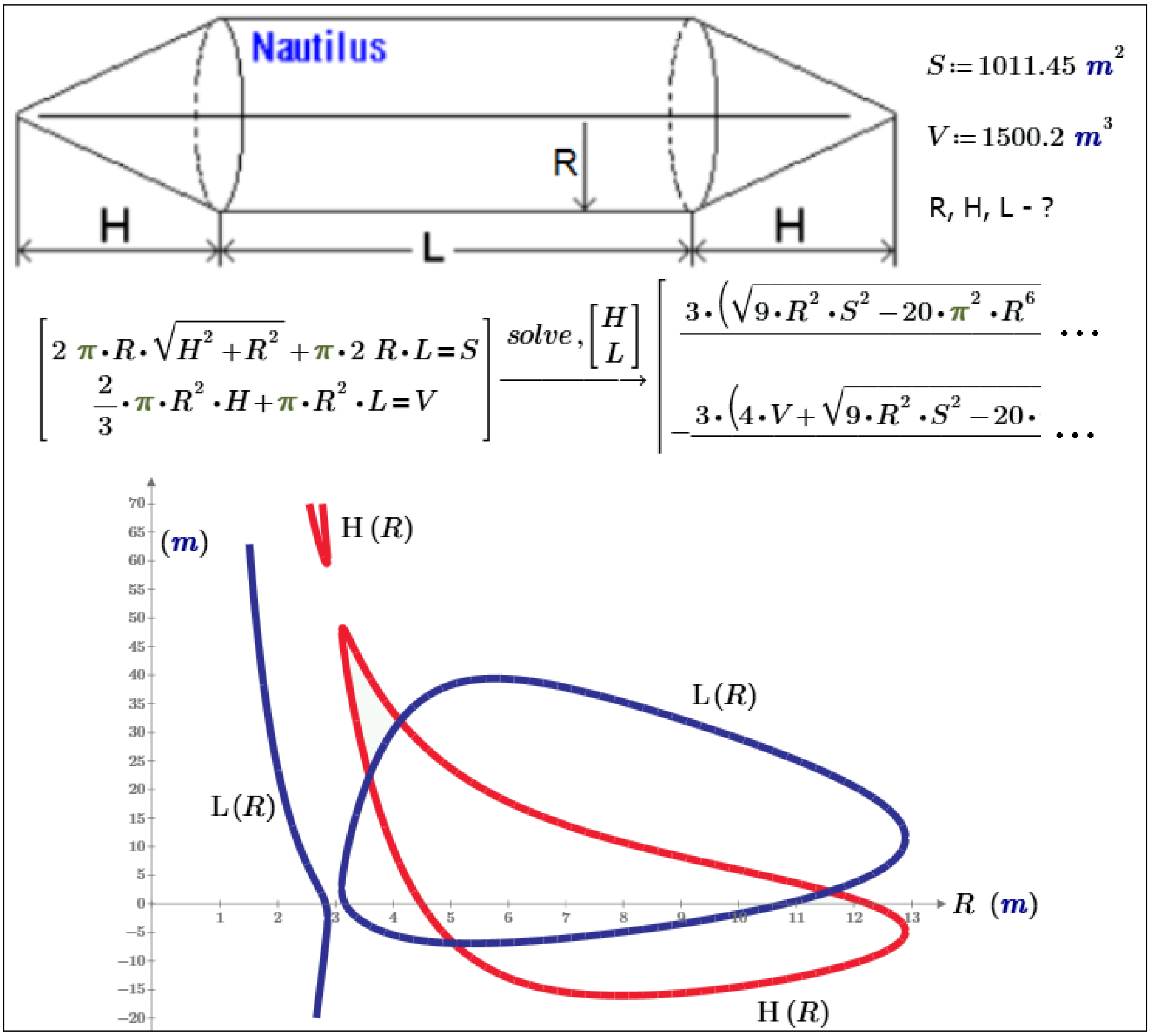


Figure 3 Jules Verne`s *mathematical problem in Mathcad*

However, Mathcad returns no specific numerical solution for this task, the system of equations is under determined. It returns a function, called Size, with the argument R. So the size of the height (H) and length (L) depend on the value of the radius (R). Besides, the system of our two non-linear equations has two solutions. Therefore, the operator returns a matrix, consisting of two lines (length L and height H) and two columns (two solutions). Due to save space, only one solution is shown in the matrix. Both solutions are displayed graphically in figure 3. It is clear, that not all solutions of the mathematical problem are also possible real solution  some of the solutions result in negative lengths and this is of course, not possible.

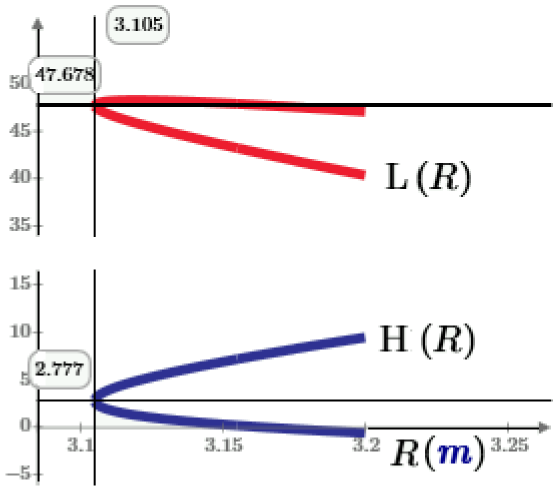


*Figure 4 solution of Jules Verne`s mathematical problem*

Assuming that the radius R of the cylindrical section of the submarine is 3.105 m, the length of its conical parts H is equal to 2,777 m and the length of the cylindrical central portion L 47,678 m. Overall length of the boat will then be equal to 53,232 m. All in all only one of the two solutions can be considered to be good. Using the “bad”-solution would results in an impractical form–the bow would be too blunt, that would increase the water resistance of the boat and did not match the image "Nautilus ", which is illustrated in the books of Jules Verne.

The Nautilus-task has much more potential for creating further tasks. Some ideas for further tasks, which can be solved by students, are given in the next paragraph.

1. Determine the minimum value of R, in which the problem has a unique solution (see left-hand end of the curves).

2. Which of the two solutions is most likely one and why?

3. Determine the radius R, for which the boat turns into a cylinder without cones (H = 0) or in two cones without a cylinder part in the middle (L = 0).

4. Determine the dimensions of the submarine with the volume = 1011,45 $m^3$ and a minimum surface.

5. Determine the dimensions of the submarine with the surface = 1500,2 $m^2$ and a maximum volume.

These are just some ideas for possible tasks, which can be solved by students. Of, course there are still a lot more possible tasks.

**MOLIÈRE`S "LE BOURGEOIS GENTILHOMME"**

DORANTE: Do you remember well all the money you have lent me?
MONSIEUR JOURDAIN: I believe so. I made a little note of it. Here it is. Once you were given two hundred louis d'or.
DORANTE: That's true.
MONSIEUR JOURDAIN: Another time, six-score.
DORANTE: Yes
MONSIEUR JOURDAIN: And another time, a hundred and forty.
DORANTE: You're right.
MONSIEUR JOURDAIN: These three items make four hundred and sixty louis d'or, which comes to five thousand sixty livres.
DORANTE: The account is quite right. Five thousand sixty livres.
MONSIEUR JOURDAIN: One thousand eight hundred thirty-two livres to your plume-maker.
DORANTE: Exactly.
MONSIEUR JOURDAIN: Two thousand seven hundred eighty livres to your tailor.
DORANTE: It's true.
MONSIEUR JOURDAIN: Four thousand three hundred seventy-nine livres twelve sols eight deniers to your tradesman.
DORANTE: Quite right. Twelve sols eight deniers. The account is exact.
MONSIEUR JOURDAIN: And one thousand seven hundred forty-eight livres seven sols four deniers to your saddler.
DORANTE: All that is true. What does that come to?
MONSIEUR JOURDAIN: Sum total, fifteen thousand eight hundred livres.
DORANTE: The sum total is exact: fifteen thousand eight hundred livres. To which add two hundred pistols that you are going to give me, which will make exactly eighteen thousand francs, which I shall pay you at the first opportunity. (Molière, 1670)

In this last example of math-lessons in classical literature are six different measuring units: louis, livre, sol, denier, pistol and franc. Although it is possible to calculate without a computer, Mathcad will be used again in order to avoid mental stress. The relationship between louis and livres is quite easy to understand: 460 louis = 5060 livres. Therefore 1 louis=11 livres. For the second equation, which can be set up, the whole text has to be quoted. It is a balance of the whole text (see Fig. 4, first line).

$$\begin{bmatrix} \begin{pmatrix} 200\cdot\text{louis} \\ +\ 120\ \text{louis} \\ +\ 140\ \text{louis} \\ +\ 1832\text{livres}\ \ldots \\ +\ 2780\text{livres}\ \ldots \\ +\ 4379\ \text{livres} + 12\ \text{sols} + 8\ \text{deniers}\ \ldots \\ +\ 1748\ \text{livres} + 7\ \text{sols} + 4\ \text{deniers} \end{pmatrix} = 15800\cdot\text{livres} \\ (200+120+140)\cdot\text{louis} = 5060\cdot\text{livres} \\ 20\ \text{sols} = \text{livre} \end{bmatrix} \text{solve}, \begin{pmatrix} \text{louis} \\ \text{livres} \\ \text{sols} \end{pmatrix} \rightarrow (2640\text{deniers}\quad 240\text{deniers}\quad 12\text{deniers})$$

$$\text{sol} = 12\ \text{deniers} \qquad \text{livres} = 20\ \text{sols} \qquad \text{louis} = 11\ \text{livres}$$

*Figure 5 MOLIÈRE`S mathematical problem in Mathcad*

In figure 5 one additional equation, which cannot be found in the text, can be seen (20 = 1 livre). Assuming, that the exchange rates have to natural numbers 20 sols have to be one livre. Other assumptions would cause non-natural numbers of the exchange rates. The correlation between pistols and francs is much more complicated. Like the "Nautilus"-task this problem is under determined. So therefore, again an assumption was made in order to have natural numbers as exchange rates. However the first line in figure 6 can be found in the text.

$$\begin{pmatrix} 15800\ \text{livres} + 200\ \text{pistoles} = 18000\ \text{francs} \\ \text{pistoles} = 11\ \text{livres} \end{pmatrix} \text{solve}, \begin{pmatrix} \text{livres} \\ \text{pistoles} \end{pmatrix} \rightarrow (\text{francs}\quad 11\text{francs})$$

$$\text{livres} = \text{francs} \qquad \text{pistoles} = \text{louis}$$

*Figure 6 MOLIÈRE`S mathematical problem in Mathcad; Part II*

Putting together both solutions the following exchange rates can be found:

1 franc = 240 deniers

1 livre = 240 deniers

1 sol = 12 deniers

1 louis = 2640 deniers

1 pistol= 2640 deniers

It can be seen, that one louis is equal to one pistol. In fact, louis and pistol are the same thing. Maybe in the future, in 100 years, scientists will also forget that “Dollars” and “Bucks” are also the same and start to calculate the exchange rate.

**SERGEJ ALEKSANDROVICH RACHINSKIJ`S “1001 TASKS FOR MENTAL CALCULATION”**

The last interesting work, which is presented in this paper, is Sergej Aleksandrovich Rachinskij`s “1001 tasks for mental calculation”. It was published in the 19th century. In our time this book was revived in the form of a website (www.1001task.ru). Owners of a smartphone can download for free problems from the book and try to solve them. All problems should be solved purely in mind, without using any utilities and/or accessories like paper, pen, calculator and so on. Rachinskij offers a lot of interesting problems in this book. It is possible to learn how people lived in the 19th century in Russia, what they did, what they sold and how high the prices were. Like in the previous tasks most problems in this book are also mainly just financial calculations, but there are also some other types of problems.

Here is one of the tasks from Rachinskij`s book:

> A coopersmith hat 8 pieces of copper, each weighing 1 pound and 8 lots. He made from these pieces some copper kettles, each of the kettles has a weight of 1 pound, 21 lots and 1 spool. How many kettles did he make? Answer: 6 (Sergej Aleksandrovich Rachinskij, 19th century)

In former times every student and even an illiterate russian person knew the correlation between pounds, lots and spools (old russian measuring units). Nowadays nobody knows anymore the correlation between these measuring units. Of course, there is the possibility just to look up the solution in the internet, but we want to calculate the correlation of these units with the help of Mathcad. Therefore the problem becomes much more difficult and also more interesting.

```
Answer := | pound ← 1
          | i ← 0
          | for n_lot ∈ 1,2..1000
          |   for n_spool ∈ 1,2..1000
          |     if 8·(pound + 8·pound/n_lot) = 6·(pound + 21·pound/n_lot + pound/(n_lot·n_spool))
          |       M<i> ← [n_lot; n_spool]
          |       i ← i + 1
          | M
```

$$Answer = \begin{bmatrix} 32 & 34 \\ 3 & 1 \end{bmatrix}$$

pound = 32 lots = 96 spools
lot = 3 spools

*Figure 7 Correlation between old measuring units in Mathcad*

Fig. 7 shows one of the possible solutions of the problem. The solution is based on trying all possible variants for the different correlations between the different measuring units (try everything for 1 pound = 1…1000lots and also 1 lot = 1…1000 spools). By the way, almost all tasks given by Rachinskij have just natural numbers. Therefore using this method for calculating the correlation between the different measuring units is legitimate.

Of course, we broke the golden rule and did not solve this problem in our mind. Instead of calculating the problem in our mind we created a special computer program for this problem. What can you say about that? Creating computer programs is now the new way of mental training. Maybe, if Rachinskij knew about computers, he would us recommend to solve these problems with the help of computers.

## CONCLUSION

This article has shown how many good examples of math-lessons in classical literature exist. In fact, there are still a lot more of interesting examples. The difficulty varies from easy to extremely difficult. Therefore, there are good challenges for each class–it is possible to integrate literature in math-classes from first class up to university. There are many good ways to integrate classical literature, instead of just reading the book, it is also possible just to watch the movie or even to act a play.

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